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On the Measurement of Dislocation Damping Forces at High Dislocation Velocity*

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ABSTRACT

In direct mobility experiments in single crystals, dislocation velocity is studied as a function of stress by the application of short-duration stress pulses. The stress pulse consists of a loading wave, followed microseconds later, by an unloading wave. At high velocities, dislocation inertia effects become important if the dislocation damping force is a decreasing function of dislocation velocity. In general, the magnitude of this force can be determined only if the relative velocity between the applied stress wave and the dislocation is considered.

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I. INTRODUCTION

During the last few years, dislocation mobility experiments have been carried out on a number of metals with close-packed atomic structures over wide ranges of temperature (from room temperature down to $4.2^{o}K$) and velocity (from 0 to approximately 1/10 of the sound velocity). The experimental work has demonstrated that interaction with thermal phonons and conduction electrons gives rise to the major source of energy dissipation for moving dislocations, and while phonon damping predominates near the Debye temperature, electron damping is strongest in the cryogenic region where atomic vibrations are weak. The dissipative force was found to be directly proportional to dislocation velocity throughout the velocity range studied. During steady-state motion, the dissipative force is equal to the driving force, and for a unit length of dislocation, is given by the familiar expression

$$Bv = \tau_R b, \tag{1}$$

where B = dislocation damping coefficient,

v = dislocation velocity,

τ_R = resolved shear stress on the dislocation,

and b = dislocation Burgers vector.

The magnitude of B is the order of 10^{-4}cgs at room temperature[1] and decreases to about 10^{-5}cgs at $4.2^{o}K$.[2] Hence the stresses required to drive dislocations at appreciable velocities in close-packed metallic structures are considerably smaller than those required in materials where thermally-activated drag mechanisms predominate (see for example, lithium fluoride,[3] or iron[4]).

Considerable speculation has centered on the question of an upper limit to dislocation velocity.[5] Continuum linear isotropic elasticity theory predicts that the total energy and effective mass of a dislocation become infinite at a velocity equal to the shear wave velocity in the material, C_t, and that therefore this velocity is an upper limit for dislocations.[6] When the discrete nature of the crystal lattice and, hence, wave dispersion, are considered, then a dislocation moving more quickly than the minimum sound velocity experiences an additional retarding force from the medium, which is very sensitive to the structure of the dislocation core.[7] Also several atomistic dislocation models have been proposed in which the dislocation energy does not diverge at C_t, but rather the lattice can support supersonic dislocation motion.[8]

Various theoretical predictions concerning dislocation velocities can be tested by experimental techniques. In the usual direct method for measuring dislocation velocity as a function of resolved shear stress, a stress pulse of known magnitude and duration is passed through a crystal and the distances individual dislocations move are observed. The stress pulse produces a resolved shear stress at the dislocation given by $\tau_R(t)$, where t is equal to time. (Note that the position of the dislocation will be changing, so that determination of $\tau_R(t)$ may be complicated.) Dislocation displacement is equal to the time integral of the dislocation velocity, or

$$d = \int v(t)dt \quad . \tag{2}$$

A relation between dislocation velocity and resolved shear stress is postulated,

$$v = f\{\tau_R\} \quad . \tag{3}$$

This expression is substituted into Eq. 2 to give a hypothetical value for displacement

$$d' = \int f\{\tau_R(t)\}\, dt, \tag{4}$$

and if d' agrees with the experimentally determined dislocation displacement, then the functional form of Eq. 3 is correct.

Implicit in Eq. 3 is the assumption that inertial effects are not important, that is dislocation velocity does not depend explicitly on time but rather is in phase with the applied stress. In Section II we shall discuss under what experimental conditions this assumption remains valid at high dislocation velocity.

The usual method of stress pulse application is by means of a loading wave followed later by an unloading wave. If the direction of stress wave propagation has a component in the direction of dislocation motion, then the relative velocities of the loading and unloading waves with respect to the dislocation must be considered to accurately determine $\tau_R(t)$ as used in Eq. 4. In Section III we shall see that this step is particularly important in the high-velocity region.

II. DISLOCATION INERTIA AND THE ACCELERATION TIME CONSTANT

In the general case, dislocation velocity is dependent explicitly on both resolved shear stress, τ_R, and time t. The functional form for this dependence is derived from the equation of motion for a long, straight dislocation,

$$\frac{d}{dt}(mv) = \tau_R(t)b - B(v)v\ , \tag{5}$$

where $B(v)$ = dislocation damping coefficient, which is some function of dislocation velocity,

m = effective mass per unit length of the dislocation.

The mass m has been calculated for both edge and screw dislocations on the basis of isotropic elasticity theory, and for a screw dislocation in uniform motion is given by its rest value divided by a velocity contraction factor.[6] That is,

$$m = \frac{\rho b^2}{4\pi\sqrt{1 - \left(\frac{v}{C_t}\right)^2}} \log_e\left(\frac{R}{r_o}\right), \tag{6}$$

where ρ = density of the medium,

R = outer cut-off radius,

and r_o = inner cut-off radius .

A similar result holds for an edge dislocation. For non-uniform motion, any change in m with velocity contains an explicit time dependence through the logarithmic term, which expresses the period required for elastic waves denoting a velocity change to radiate from the dislocation core.[9]

It is useful to consider the motion of a dislocation, initially moving with uniform velocity v_o, caused by a sudden small change in applied stress. Then the response to an arbitrary stress change can be found by integration of a series of impulsive changes in stress. In the limit of small velocities, $v << C_t$, m reduces to its rest value, m_o, and $B(v)v$ is equal to Bv. Eq. 5, with $\tau_R(t)$ equal to τ_R integrates quite simply to

$$v = v_o + \frac{\tau_R b}{B}\left(1 - \exp\left[-\frac{B}{m_o}t\right]\right) . \qquad (7)$$

The acceleration time constant, m_o/B, is of the order of 10^{-10} to 10^{-11} sec at room temperature for close-packed metals and increases by about one order of magnitude at 4.2^oK. Typically in a dislocation mobility experiment, the applied stress increases from 0 to its maximum value in a time between 10^{-6} and 10^{-5} sec, so that an acceleration time constant less than $\sim 10^{-7}$ sec implies that dislocation velocity is effectively in phase with the applied stress, as in the case above.

In the region of large dislocation velocity, the velocity contraction factor in the expression for m becomes important. However, even at cryogenic temperatures, provided that B(v) does not decrease with dislocation velocity, m/B(v) remains less than 10^{-8} sec for v as large as $0.99C_t$. Hence inertial effects, and the explicit time dependence of v, become important only if B(v) decreases as the dislocation velocity increases ($v \leq 0.99\, C_t$).

III. DISLOCATION MOTION CAUSED BY AN IMPINGING STRESS WAVE

In the following, we calculate the stress-time history $\tau_R(t)$, for a long, straight dislocation responding through its motion to an impinging stress wave. As an illustrative example in the analysis, we utilize a linear relation between velocity and stress (Eq. 1), but any functional form may be treated by the simple method developed.

A plane stress wavefront, moving in a direction $\underline{x}$ in a crystal is described by its wave velocity, $\underline{C}$, and stress gradient at any point $\frac{\partial \tau}{\partial t}\big|_x$. Assuming that the wave is non-dispersive and the crystal perfectly elastic, we can write

$$\tau(x,t) = \tau(x \pm Ct) ,$$
$$\frac{\partial\tau}{\partial t}\Big|_x = \pm C \frac{\partial\tau}{\partial x}\Big|_\tau , \tag{10}$$

where the negative sign refers to a wave travelling in the positive x-axis direction.

Consider a dislocation slip direction, $\underline{x}'$, at an angle θ to the direction of propagation of the wavefront, and lying in the plane (x, x') defined by the normal direction $\underline{y}$ (Fig. 1). A long, straight dislocation moves with velocity $\underline{v}$ in the positive $\underline{x}'$ direction (the slip plane is defined by the vector $\underline{x}' \times \underline{y}$). Let the $\underline{x}''$ direction be fixed to the dislocation and parallel to $\underline{x}'$. Then the x-coordinate of the dislocation is given by

$$x = \cos\theta \left[x'' + \int_0^t v(t')dt' \right], \; v(0) = 0. \tag{11}$$

From Eq. 10 we find

$$\tau(x'',t) = \tau\left(x''\cos\theta + \cos\theta\int_0^t v(t')dt' \pm C(t)\right), \tag{12}$$

and

$$\frac{\partial\tau}{\partial t}\Big|_{x''} = \left(\frac{v\cos\theta}{C} \pm 1\right)\frac{C}{\cos\theta}\frac{\partial\tau}{\partial x''}\Big|_t . \tag{13}$$

But

$$\frac{\partial\tau}{\partial x''}\Big|_t = \frac{\partial\tau}{\partial x}\Big|_t \cos\theta , \tag{14}$$

so using Eq. 10, Eq. 13 becomes

$$\frac{\partial\tau}{\partial t}\Big|_{x''} = \pm\left(\frac{v\cos\theta}{C} \pm 1\right)\frac{\partial\tau}{\partial t}\Big|_x . \tag{15}$$

Let the dislocation velocity be related to the applied stress at $x'' = 0$ by some continuous function, $v = f\{\tau\}$, as in Eq. 3. Substituting this function into Eq. 15, the differential relation for the stress-time history of the dislocation is

$$\frac{d\tau}{\pm\left[\frac{f\{\tau\}\cos\theta}{C} \pm 1\right]} = \frac{\partial\tau}{\partial t}\Big|_x dt\ . \tag{16}$$

Eq. 16 is perfectly general in that its integration depends solely on the functional forms chosen for $f\{\tau\}$ and the stress gradient in the shear wave $\frac{\partial\tau}{\partial t}\big|_x$.

In the remainder of this section, we investigate the case of a dislocation accelerating from rest under the influence of a rising stress wave, and constrained by geometry to move in the positive $\underline{x}'$ direction, and subsequently decelerated by an equivalent unloading wave. We choose

$$\frac{\partial\tau}{\partial t}\Big|_x = \pm\ k\ , \tag{17}$$

where k = constant, and the positive sign refers to the accelerating wave. This corresponds closely to the situation realized in the usual experiment. Finally, we assume that a linear relation between velocity and resolved shear stress holds through the velocity range, that is

$$v = \frac{\alpha b}{B}\tau\ , \tag{18}$$

where α is the shear stress-resolving factor. An upper limit for k is imposed by the requirement that the duration of time, from rest to a maximum velocity, be of the order of 10^{-6} sec or greater. For B equal to

10^{-5} cgs and a maximum velocity of 2×10^{5} cm/sec, then k must be about 10^{14} dynes/cm^{2}sec or less.

For the accelerating dislocation, Eq. 16 becomes

$$\frac{d\tau}{\left[1 - \dfrac{\alpha b\tau\cos\theta}{BC}\right]} = k dt \ , \tag{19}$$

and integrating

$$\tau = \frac{BC}{\alpha b\cos\theta}\left(1 - \exp\left[-\frac{\alpha bk\cos\theta}{BC}t\right]\right) . \tag{20}$$

The dislocation velocity is

$$v = \frac{C}{\cos\theta}\left(1 - \exp\left[-\frac{\alpha bk\cos\theta}{BC}t\right]\right) , \tag{21}$$

and its displacement, d, is found from

$$\begin{aligned} d &= \int_o^t v dt \\ &= \frac{C}{\cos\theta}t + \frac{Bc^2}{\alpha bk\cos^2\theta}\left(\exp\left[-\frac{\alpha bk\cos\theta}{BC}t\right] - 1\right) . \end{aligned} \tag{22}$$

According to Eq. 21, the dislocation velocity approaches an upper limit $C/\cos\theta$ exponentially, at a rate determined by the time constant $\frac{BC}{\alpha bk\cos\theta}$. This limit of velocity is obvious, conceptually, for a dislocation can experience no change in the strength of a disturbance when it is moving with its velocity of propagation, C. In the case of a crystal, the upper limit of dislocation velocity is not necessarily $C/\cos\theta$, which increases without limit as $\theta \to \pi/2$. The approach we employed here is simply an analysis of relative motion and does not include the limitations imposed upon dislocation velocities in solids. These, in turn, impose an upper limit in time for the integrability of Eq. 19.

By combining Eqs. 21 and 22, a simple expression for the distance, d_F, travelled by the dislocation in order to reach a certain fraction of the wave velocity is found. If we let $v/C = F$, then

$$d_F = \frac{BC^2}{\alpha bk\cos^2\theta}\left[\log_e\left(\frac{1}{1 - F\cos\theta}\right) - F\cos\theta\right]. \tag{23}$$

To conclude this section we consider a dislocation initially moving with velocity $\underline{v}_m$ in the positive $\underline{x}'$-direction and interacting with an unloading wave moving in the negative $\underline{x}$-direction. In this case, Eq. 16 reduces to

$$\frac{d\tau}{\left[\frac{\alpha b\tau\cos\theta}{BC} + 1\right]} = -kdt \quad , \tag{24}$$

and integrating,

$$\tau = \frac{BC}{\alpha b\cos\theta}\left\{\left(1 + \frac{v_m\cos\theta}{C}\right)\exp\left[-\frac{\alpha bk\cos\theta}{BC}t\right] - 1\right\} , \tag{25}$$

$$v = \frac{C}{\cos\theta}\left\{\left(1 + \frac{v_m\cos\theta}{C}\right)\exp\left[-\frac{\alpha bk\cos\theta}{BC}t\right] - 1\right\} , \tag{26}$$

and

$$d = \frac{BC}{\alpha bk\cos^2\theta}\left(1 + \frac{v_m\cos\theta}{C}\right)\left(1 - \exp\left[-\frac{\alpha bk\cos\theta}{BC}t\right)\right) - \frac{C}{\cos\theta}t . \tag{27}$$

By combining Eqs. 26 and 27, we arrive at an expression for the distance necessary to bring a dislocation to rest from a fraction F of the shear wave velocity. It is

$$d_F = \frac{BC^2}{\alpha bk\cos^2\theta}\left[F\cos\theta - \log_e(1 + F\cos\theta)\right] . \tag{28}$$

IV. DISCUSSION

We have treated the most simple case of an advancing plane stress wavefront on dislocation motion, that in which $\frac{\partial\tau}{\partial t}\big|_x$ is constant and dislocation velocity is proportional to applied shear stress. Any general form for $\frac{\partial\tau}{\partial t}\big|_x$ or stress-dependence of velocity $f\{\tau_R\}$ can be handled, although closed-form solutions may not be obtainable in all cases. The example above, however, is sufficient to point out the importance of considering relative motion when $v = f\{\tau_R\}$ is to be determined experimentally.

As a numerical illustration, we computed the distance required to accelerate a dislocation to some fraction F of the wave velocity and then bring it to rest, by applying both the approach outlined above, using Eqs. 23 and 28, and the classical method where relative motion is ignored and the distance is simply $\frac{BC^2}{\alpha bk}F^2$. The two cases are compared in Fig. 2 for $\theta = 0$. (In the limit $\theta = \pi/2$, the two converge.) The distance computed from the first approach begins to diverge noticeably from that of the second when F reaches about 0.5. The former case gives a result similar to that when relative motion is ignored but the velocity-stress relation is assumed to be non-linear with the form

$$\frac{B}{\left[1 - \left(\frac{v}{c}\right)^2\right]^n} v = \alpha\tau b , \tag{29}$$

where n is a constant greater than zero. (The case $n = \frac{1}{2}$ is plotted in Fig. 2.) Therefore if relative velocity between the dislocation and applied stress wave is neglected in the analysis of experimental data, then a non-linear damping coefficient will be deduced that is in error by a "relativistic" factor of the form $\left[1 - \left(\frac{v}{c}\right)^2\right]^n$.

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REVISED FIGURE CAPTION

Figure 1. Dislocation gliding in the slip plane defined by the normal vector $\underline{x}' \times \underline{y}$. The slip direction $\underline{x}'$ is at an angle θ from the direction of propagation of the stress wave.